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By Universitas Muhammadiyah Sidoarjo

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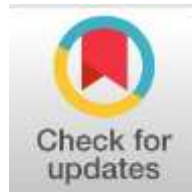
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Guided Discovery Learning And Fraction Puzzles Facilitate Mathematical Comprehension For Slow Learners: Pembelajaran Penemuan Terbimbing Dan Teka Teki Pecahan Memfasilitasi Pemahaman Matematis Siswa Lamban Belajar

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Abstract

General Background Primary education establishes foundational mathematical competencies necessary for logical reasoning and quantitative literacy. **Specific Background** Students at risk of being slow learners often struggle with abstract mathematical topics, particularly proportional quantities, due to limited working memory capacity and slower cognitive processing speeds. **Knowledge Gap** Traditional mathematics instruction predominantly utilizes abstract, lecture-based procedures without sufficient concrete manipulative scaffolding, thereby overwhelming the cognitive load of slower learners. **Aims** This research investigates the application of a modified guided pedagogical framework supported by physical manipulative media to facilitate mathematical conceptualization in fifth-grade students facing cognitive delays. **Results** Following a two-cycle adaptive classroom action research methodology, the participants' average conceptual understanding scores progressed from 48.0 to 78.0. Furthermore, classroom mastery rates grew from 0% in the preliminary phase to 100%, alongside notable increases in active participation and independent problem-solving capabilities. **Novelty** This study uniquely adapts exploratory pedagogical frameworks by integrating visual simplifications and concrete enactive manipulation to specifically accommodate the restricted working memory of slower processing students. **Implications** Providing structured, hands-on learning experiences offers a practical strategy for educators to minimize extraneous cognitive load and cultivate proportional reasoning in inclusive elementary classrooms.

Highlights

- The modified pedagogical intervention progressed the average participant test scores from 48.0 to 78.0 across two instructional cycles.
- Classroom mastery rates for proportional reasoning tasks advanced from zero to full attainment following the concrete manipulative integration.
- Participants demonstrated substantial behavioral transitions from passive observation to active engagement and independent cognitive processing.

Keywords

Proportional Reasoning; Cognitive Load; Inclusive Classrooms; Concrete Manipulatives; Action Research

INTRODUCTION

Education is a basic element of human development that constantly develops with social, economic and technological changes. Education is a way to transfer knowledge but also as a systematic process to develop a person's cognitive, affective, and psychomotor competencies to participate effectively in society (Restian & Widodo, 2019). In the process of learning through educational experiences, learners gain knowledge, skills, values and attitudes, which allow them to solve problems and adapt to changing environments. In this context, learning is a conscious process of designing learning conditions that support the achievement of curriculum objectives and the experience of meaningful learning (Ismunarti et al., 2020). Thus, the improvement of classroom instruction quality is still one of the main concerns in elementary education. Primary education is very important in laying the groundwork for students' intellectual, emotional, and social development. At this stage of schooling students begin to develop the logical reasoning, conceptual understanding and problem-solving skills that will support their learning at higher levels of education. Mathematics is one of the core subjects in elementary schools. It helps students develop analytical thinking, logical reasoning, and quantitative literacy. The mathematics learning is generally described as a process in which students progressively build new knowledge by linking previously acquired concepts with new mathematical ideas. However, mathematics is often considered one of the most difficult subjects by elementary school students despite its importance because many mathematical concepts are abstract and require higher-order cognitive processing.

According to Piaget's theory of cognitive development, many elementary school children are in the concrete operational stage. This means they learn best with concrete objects and hands-on experiences, rather than with abstract symbols. Therefore, students may have difficulty learning mathematics with an overreliance on symbolic representation without adequate concrete visualization. In mathematics education, conceptual understanding is defined as students' ability to understand the meaning, attributes, and concepts of mathematics and to use appropriate procedures in solving mathematical problems (Dewanti, 2024). Students who lack conceptual understanding tend to memorize procedures mechanically rather than understanding the relationships among mathematical concepts, which leads to persistent misconceptions and low achievement. For students who are in danger of becoming slow learners, the difficulties are far greater. Students in this group tend to have slightly below average intellectual abilities, generally with intelligence quotient (IQ) scores between about 70 and 85, but not classified as having intellectual disabilities. They generally have a slower rate of processing information, a smaller capacity of working memory, a shorter attention span and more difficulty in applying previously learned knowledge to new situations. Consequently, they need further instruction, more practice, concrete learning experiences, and individualized coaching to reach learning targets similar to their peers. However, in inclusive elementary classrooms these students typically receive the same instructional approach as other students, even though their learning characteristics and educational needs are significantly different.

Fractions is one of mathematical topic that always give a lot of learning difficulties for students that are potential slow learners. Fractions are one of the basic mathematical concepts because they are a prerequisite for learning ratios, proportions, decimals, percentages, algebra, and many other mathematical topics encountered in higher grades (Putri, 2022). As students transition from whole numbers to fractions, they learn to think about part-whole relationships. Students need to realize that the numerator and denominator work together to represent one number, not two different numbers. This conceptual understanding is cognitively demanding because it requires proportional reasoning and relational thinking, both of which are substantially more abstract than counting whole numbers. In many studies, misconceptions of fractions have been reported as one of the most persistent mathematical learning difficulties experienced by elementary school students. Such misconceptions are greater among students who are at risk for being slow learners. For example, students often think that one-fourth ($\frac{1}{4}$) is greater than one-half ($\frac{1}{2}$) because 4 is bigger than 2. Others wrongly add the numerators and denominators directly to do the addition of the fractions (e.g., they might say that $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$). Such misconceptions are indicative of student tendency to apply whole-number reasoning, instead of understanding fractions as representing proportional quantities. If these misconceptions are not corrected early on, they may influence the students' understanding of later mathematical concepts.

Preliminary observations and diagnostic assessment in Grade V at SDN 009 Kombeng found five students who are at risk of becoming slow learners. Common learning characteristics of these students were difficulty in following classroom instructions, slow responses during mathematical activities, limited retention of procedural steps, and significant difficulties in understanding fraction concepts. Their pre-cycle achievement score averaged 48.00, well below the school's Minimum Mastery Criterion (MMC). In addition, classroom observations showed that these students seldom participated actively in mathematics lessons, were often dependent on teacher help, and lacked confidence when they attempted to solve mathematical problems independently. The instructional practices implemented in the classroom were also linked to the learning difficulties of these students. The teaching of mathematics was mainly teacher-centered, emphasizing direct explanation and symbolic procedures with few opportunities for students to engage with concrete learning materials. Under these instructional conditions, students were expected to understand mathematical symbols before they had developed adequate representations conceptually. This type of instructional approach is especially difficult for students who are at risk of being slow learners as their cognitive characteristics necessitate a gradual transition from concrete experiences to abstract reasoning. This causes these students to become passive learners, get frustrated with class activities and gradually lose interest in learning mathematics.

One instructional model that can overcome these challenges is Discovery Learning. The model of instruction promotes active construction of knowledge by students through exploration, observation, investigation, and derivation of conclusions from the learning experiences. The discovery process is meant to generate deeper conceptual understanding in students, since knowledge is actively constructed not passively received. However, prior educational research has shown that pure

Discovery Learning may not be effective for students with limited cognitive capacity as the instructional demands may exceed their working memory capacity. According to Cognitive Load Theory, excessive cognitive demands can overwhelm students' limited processing capacity, thus hindering rather than promoting learning effectiveness. Therefore the use of Discovery Learning to students who are at risk of being slow learners needs to be modified in teaching appropriately. Instead of expecting students to discover concepts on their own, teachers should systematically scaffold learning through providing simplified instructions, carefully sequenced learning activities, guided questioning, additional time for learning, and continuous formative feedback. The guided approach helps students to engage in the discovery process, and reduces unnecessary cognitive load. Thus, the learning process is more accessible to slower cognitive processing students without eliminating the necessary features of Discovery Learning.

Besides instructional scaffolding, it is also important to use manipulative learning media to facilitate conceptual understanding of students at risk of being slow learners. One of the instructional media that is considered very appropriate for learning fractions is the Fraction Puzzle. Fraction Puzzle media are made from durable materials such as EVA foam or thick cardboard. Students can touch fraction pieces, compare equivalent fractions, and see how fractions add up to a whole through hands-on activities. Students move through the enactive stage of learning, what Bruner calls putting together, matching, and combining puzzle pieces, before moving to pictorial and symbolic representations. Such concrete learning situations reduce abstraction, consolidate conceptual knowledge, and allow students to visualize mathematical relations that are often hard to grasp when taught only symbolically. This classroom action research is based on the theoretical basis and empirical problems that occur in the preliminary observation to improve students' conceptual understanding of fractions through the application of the Discovery Learning model assisted by Fraction Puzzle media for fifth grade students who are at risk of being slow learners at SDN 009 Kombeng. The results of this study are believed to contribute to the development of successful teaching strategies for mathematics education in inclusive elementary classrooms and to provide practical recommendations for teachers to improve students' conceptual understanding with different learning characteristics.

METHOD

This study used Adaptive Classroom Action Research (CAR) based on the spiral model proposed by Kemmis and McTaggart (1988) which consists of four stages, namely planning, action, observation, and reflection. The research was conducted in two action cycles. Each action cycle included the instructional improvements based on the reflection of the previous cycle. The research was conducted at SDN 009 Kombeng, East Kutai Regency, Indonesia during the second semester of the 2025/2026 academic year. The participants were five fifth grade students who were identified as at-risk of becoming slow learners based on diagnostic assessment, classroom observations and mathematics achievement records. The students demonstrated characteristics such as slow processing of information, short attention spans, and consistently low achievement in mathematics, particularly on topics involving fractions. The intervention was done by applying Discovery Learning model assisted by Fraction Puzzle manipulative media. Students were guided through the learning process, from the stages of stimulation, problem identification, data collection, data processing, verification, and generalization, with systematic instructional scaffolding according to their learning characteristics. The Fraction Puzzle media enabled students to manipulate concrete representations of fractions to foster conceptual understanding of equivalent fractions and addition of fractions with common denominators. Data were collected through written tests, observation of the classroom, clinical interviews and documentation. The conceptual understanding of fractions was assessed by written tests that were given before the intervention (pre-test), at the end of Cycle I, and at the end of Cycle II. Classroom observation was conducted using structured observation sheets to evaluate students' engagement and participation in Discovery Learning activities. Clinical interviews were individually conducted after each learning cycle to identify students' conceptual understanding, misconceptions, and learning experiences. Expert judgment was used to review the research instruments for content validity, and methodological triangulation was employed to enhance the credibility of the findings by comparing data from tests, observations, interviews, and documentation. The quantitative data were analyzed descriptively by calculating students' mean scores and the percentage of learning mastery. Based on the characteristics of the students at risk of becoming slow learners, the modified minimum mastery criterion was set at a score of 65. The classroom action research was said to be successful if at least 80% of the participants (four out of five students) achieved the minimum mastery criterion and showed the improvement of engagement and active participation during the learning process.

RESULT AND DISCUSSION

Initial Condition (Pre-Cycle)

The results of the pretest conducted in early May 2026 on five fifth-grade students at SDN 009 Kombeng identified as at risk of becoming slow learners showed that all of them had significant difficulty in understanding the concept of fractions. The students posted a mean score of 48.00 which was below the modified Minimum Mastery Criterion (MMC) of 65 resulting to a learning mastery rate of 0%. These findings indicate that the students' initial understanding of equivalent fractions and the addition of fractions with common denominators was still insufficient. Classroom observations also showed that the limited conceptual understanding was linked to instructional practices that were predominantly based on teacher explanations and abstract mathematical symbols. The learning activities provided few opportunities for students to engage with concrete learning materials that could support visual representations of fraction concepts. Thus, the students who were at risk of becoming slow learners had trouble associating visual representations with the notation of the fractions and the relationship between the numerator and denominator as part of one mathematical concept.

An analysis of students' responses revealed a number of common misunderstandings. DR thought $\frac{1}{4}$ was bigger than $\frac{1}{2}$ because four is a bigger number than two. Student IK had difficulty distinguishing between the numerator and denominator

when interpreting a fraction picture. Student MA added both numerators and denominators incorrectly, finding that $1/4 + 2/4 = 3/8$. Student NA had trouble with reading comprehension and was unable to represent fractions using correct mathematical notation. Meanwhile, student ZAG often forgot the procedural steps needed to add fractions, and showed passive learning behavior, often giving up when solving mathematical problems. Altogether, the pre-cycle results indicate that the participants did not have enough conceptual understanding of fractions. These findings underscore the significance of an instructional intervention that provides concrete, structured, and learner-centered learning experiences that are compatible with the characteristics of students at risk of being slow learners. So these initial results became the basic foundation for implementing the first cycle of classroom action research with the Discovery Learning model assisted by Fraction Puzzle media.

Implementation and Results of Cycle I

The first cycle was carried out for two meetings in the fifth grade classroom of SDN 009 Kombeng in mid-May 2026. The instructional intervention applied the Discovery Learning model modified with intensive instructional scaffolding to suit the learning characteristics of students at risk of slow learners. The learning process applied six steps of Discovery Learning, namely stimulation, problem statement, data collection, data processing, verification, and generalization. In the Stimulation Stage, the teacher presented a sponge cake as a concrete representation of the part to whole concept and then introduced the colorful Fraction Puzzle media made from EVA foam. Students were prompted to look at the cake and respond to guiding questions such as, "What would happen if this cake was cut into two equal pieces?" and "Can two smaller pieces make one larger piece?" The activities were designed to activate students' prior knowledge before the introduction of the concepts of fractions. During the problem statement stage, students were asked to find the relationship between fractions by simple questions from the teacher. In the later stages of data collection, the five students who were at risk of being slow learners were clustered together to allow for intensive one-to-one scaffolding. Students manipulated, built, compared, and stacked the Fraction Puzzle pieces to discover for themselves that two $1/4$ pieces covered one $1/2$ piece exactly, and experienced the notion of equivalent fractions first hand. Then they investigated adding fractions with common denominators by putting together puzzle pieces and then writing down what they found using mathematical symbols. During the data processing phase, students filled out worksheets with the results of their exploration. The teacher provided step-by-step guidance for students who had difficulty making connections between the concrete manipulative activities and the symbolic mathematical representations. In the verification stage, the answers of students were discussed collectively to verify the conceptual accuracy, and in the generalization stage, students were guided to draw simple conclusions about equivalent fractions and the addition of fractions with common denominators according to their individual abilities.

Based on the observation in the classroom, the use of Fraction Puzzle media can increase students' attention and involvement in the learning process. Students were more enthusiastic than before the cycle because they could directly handle, assemble, compare and manipulate the puzzle pieces. Hands-on learning activities provided students opportunities to develop conceptual understanding through concrete experiences, which boosted participation in classroom discussions and a willingness to work on math problems. However, there were still some challenges in Cycle I. During data processing, students NA, IK, and DR continued to struggle to translate their manipulative experiences into symbolic mathematical representations. These students required more time to understand each procedural step, as the written instructions of the worksheets were relatively long and generated a considerable cognitive load. Moreover, the time provided for exploration in Cycle I was not enough for some students who have a slower pace of information processing and need more time to learn to fully understand the concepts.

Table 1. Results of the Cycle I Achievement Test

No.	Student Initials	Pre-Cycle Score	Cycle I Score	Learning Status	Teacher's Observation
1	DR	50	65	Mastered	Began to understand the concept of equivalent fractions through manipulation of the Fraction Puzzle and was able to distinguish fractions based on area representation.
2	IK	45	58	Not Yet Mastered	Successfully assembled the puzzle pieces but remained hesitant when converting visual representations into fraction symbols.
3	MA	55	68	Mastered	Successfully understood the addition of fractions with common denominators and no longer added the denominators together.
4	NA	40	55	Not Yet Mastered	Required more time to understand worksheet instructions and continued to need intensive teacher guidance.
5	ZAG	50	74	Mastered	Demonstrated high enthusiasm while using the Fraction Puzzle, actively participated during discussions, and showed substantial improvement in learning motivation.
Mean Score		48.00	64.00		Cycle I Learning Mastery = 60% (3 of 5 students achieved mastery)

As shown in Table 1, the students' average achievement score rose from 48.00 in the pre-cycle assessment to 64.00 at the end of Cycle I. 60% (3 out of the 5 participants) obtained the modified Minimum Mastery Criterion (MMC) of 65. The remaining two students did not achieve the expected level of achievement. The highest improvement was achieved by student ZAG who scored 74 points followed by student MA with 68 points. Students IK and NA, meanwhile, continued to need additional instructional support, especially in linking concrete representations and symbolic notation and in understanding written instructions independently. The results of Cycle I in general indicate that the application of the Discovery Learning model helped by Fraction Puzzle media has a positive effect on improving students' conceptual understanding of fractions for students who are slow learners. The intervention also increased students' motivation, attention, classroom participation, and desire to explore mathematical ideas, in addition to improving learning outcomes. Overall, the mastery rate was only 60%, which did not meet the pre-determined success criterion that at least 80% of participants should obtain a minimum score of 65. Based on the reflections of Cycle I, several instructional improvements

were anticipated for Cycle II. These were: (1) simplifying worksheets by increasing visual representations and decreasing written instructions, (2) extending exploration time with the Fraction Puzzle media, (3) providing more intensive individualized scaffolding for students IK and NA, and (4) implementing gradual practice activities before students engaged in independent problem-solving. These improvements were expected to further enhance students' conceptual understanding and the study meet its predetermined success criteria in Cycle II.

Implementation and Results of Cycle II

The reflection after Cycle I showed that students at risk of being slow learners still had difficulties because of their low working memory capacity and slow speed of information processing. The implementation of the Fraction Puzzle had improved conceptual understanding but some students struggled to translate their concrete learning experiences into symbolic mathematical representations. The teacher also noted that the worksheets had too many written instructions, which contributed to students' cognitive load and reduced their capacity to focus on key mathematical concepts. Based on these results, several instructional modifications were made during Cycle II, which was done in late May 2026. First, the student worksheets were redesigned by increasing the number of visual illustrations while substantially reducing written explanations to minimize unnecessary cognitive demands. Second, students had more time to explore and collect data so they could play with the Fraction Puzzle multiple times before moving to written tasks. Third, the teacher provided students with immediate feedback when they put the puzzle pieces together correctly or incorrectly, so that misconceptions could be corrected as they arose. Finally, a "point-and-say" instructional technique was employed in which students were asked to point to each puzzle piece while simultaneously verbalizing its corresponding fraction notation. This strategy was designed to strengthen the connection between concrete visual representations and abstract mathematical symbols.

The instructional enhancements used in Cycle II resulted in significant improvements in students' learning behaviors and conceptual understanding. Classroom observations indicated that all five students showed higher levels of confidence, attention span, and active participation during the Discovery Learning process. More time to explore meant that students were able to play with the Fraction Puzzle without feeling rushed, while visual worksheets that were less complex reduced the cognitive load of reading long written instructions. Students also demonstrated significant improvement in their ability to communicate mathematically. And as the discussions continued, they became more and more willing to answer questions, explain their reasoning, and check their solutions using the puzzle media. Students IK and NA started to overcome misconceptions seen before through repeated guided practice and individualized scaffolding. Immediate corrective feedback also increased students' confidence and reduced their hesitation when solving fraction problems independently.

Table 2. Results of the Cycle II Achievement Test

No.	Student Initials	Pre-Cycle Score	Cycle I Score	Cycle II Score	Learning Status	Development of Conceptual Understanding / Interview Findings
1	DR	50	65	75	Mastered	Demonstrated a solid understanding that $\frac{1}{2} = \frac{2}{4}$ after repeatedly stacking equivalent puzzle pieces.
2	IK	45	58	70	Mastered	Successfully solved $\frac{1}{6} + \frac{3}{6} = \frac{4}{6}$ fluently using the Fraction Puzzle as a visual aid.
3	MA	55	68	85	Mastered	Worked independently and accurately solved contextual fraction problems involving common denominators.
4	NA	40	55	78	Mastered	Successfully overcame slow information processing through intensive scaffolding and visually simplified worksheets.
5	ZAG	50	74	82	Mastered	Actively explained the concept of equivalent fractions to classmates during the verification stage, indicating deeper conceptual understanding.
Mean Score		48.00	64.00	78.00		Cycle II Learning Mastery = 100% (5 of 5 students achieved mastery)

The students' mean achievement for Cycle I was 64.00 while for Cycle II the students' mean achievement was 78.00. The overall improvement was 30 points as compared to the pre-cycle assessment as shown in Table 2 . Most importantly, all five participants (100%) met the modified Minimum Mastery Criterion (MMC) of 65, indicating that the predetermined success criterion of the classroom action research had been fully met. There were also significant gains in conceptual understanding in the individual trajectories of each participant. Repeated manipulation of the puzzle pieces led to a stable understanding of equivalent fractions for student DR. For example, student IK, who previously had difficulty translating visual representations into symbolic notation, was able to successfully and confidently perform fraction addition. Student MA increased independence in solving contextual fraction problems without teacher help. Student NA showed impressive growth after receiving intensive individualized scaffolding and working with visually simplified worksheets. Student ZAG not only mastered the learning objectives, but was also able to explain equivalent fraction concepts to peers during classroom discussions, indicating a greater level of conceptual understanding. In general, the findings of Cycle II suggest that the instructional improvements made after the reflection stage have greatly improved both the cognitive and affective learning outcomes of students who are at risk of being slow learners. The integration of Discovery Learning model with Fraction Puzzle manipulative media, individual scaffolding, simple visualization worksheets, extended exploration time, and immediate feedback fostered a conducive learning environment that effectively supported the conceptual understanding of fractions. All participants achieved the predetermined mastery criteria, and they are actively involved in the learning process. Therefore, the classroom action research was stopped after Cycle II without any additional intervention cycles.

Discussion

The research results indicate that the application of the modified Guided Discovery Learning model with Fraction Puzzle media can improve the conceptual understanding of fractions for fifth grade students at risk of being slow learners at SDN

009 Kombeng. The same improvement also happened in the students' mean score, from 48.00 in the pre-cycle to 64.00 in Cycle I and 78.00 in Cycle II. The percentage of students who achieved learning mastery also increased from 0% in the pre-cycle to 60% in Cycle I and 100% in Cycle II. These results indicate that the instructional intervention was effective in facilitating students' understanding of equivalent fractions and addition of fractions with common denominators. Prior to the intervention, the students found it difficult to understand fractions as relating parts to a whole. Many students thought that $\frac{1}{4}$ was bigger than $\frac{1}{2}$ because 4 was a bigger number than 2. Other students added the numerators and the denominators when solving problems of addition of fractions. These mistakes indicate that the students tended to regard the numerator and denominator as whole numbers rather than as parts of a fraction. Such misconceptions also suggest that abstract explanations and symbolic procedures alone were insufficient to meet the learning needs of students at risk of being slow learners. The use of the Fraction Puzzle media helped the students to change the abstract fraction concepts to the concrete and observable experiences. Students had opportunities to hold, arrange, compare, combine and overlay fraction pieces. For example, students showed the equivalence of $\frac{1}{2}$ and $\frac{2}{4}$ by stacking two quarter pieces over one half-piece. Since the two arrangements occupied the same area, students could see that the two fractions represented the same amount. Through this activity they were able to develop conceptual meaning from direct experience rather than just memorizing math rules.

The result confirms the Bruner's representation theory. Bruner describes learning to grow through the enactive, iconic and symbolic stages (Bruner, 1966). The enactive stage of learning occurs when students learn through direct manipulation of concrete objects. In the iconic stage, visual images and representations help understanding, while in the symbolic stage students use numbers and mathematical notation. In the present study, the enactive stage was the manipulation of Fraction Puzzle pieces, the iconic stage was the visual illustrations in the worksheets, and the symbolic stage was the use of fraction symbols. The students understood the fractions in a more meaningful way through the gradual change through these stages. The Fraction Puzzle media also assisted students in understanding the concept of fraction addition. In the pre-cycle, some students added the numerators and denominators and incorrectly calculated $(\frac{1}{4} + \frac{2}{4})$ as $(\frac{3}{8})$. The students combined one one-quarter puzzle piece with two other one-quarter puzzle pieces and discovered that they made three pieces of the same size. So they began to see that the numerator is the number of equal parts that are being added together and the denominator is the fixed size of each part. The hands-on activity allowed students to see why the numerator changes but the denominator is the same when adding fractions with common denominators. The results of Cycle I indicated that the use of manipulative media improved students attention and participation. The average score improved by 16 points from 48.00 to 64.00. Three out of five students met the mastery criterion. But the results had not reached the expected success indicator because two students were still below the minimum mastery criterion. Classroom observations indicated that some students were able to manipulate the puzzle pieces successfully but struggled to transform the concrete representations to fraction symbols. They also needed more time to read the worksheets, to understand the directions, and to write their responses.

These challenges suggest that manipulative media alone do not necessarily enhance conceptual understanding. Media effectiveness is assessed by the extent of its integration into the instructional process. In Cycle I several instructions written on the student worksheets were still relatively complicated. Students had to read, interpret pictures, assemble the puzzle pieces, and write mathematical symbols all at the same time. The demands for simultaneous processing increased the amount of information that had to be processed via working memory. This condition can be explained by Cognitive Load theory which states that working memory has a limited capacity and learning may be disrupted when learners are required to process excessive information at the same time (Sweller et al., 2019). Students who are at risk for being slow learners generally need more time to process information, to understand directions, and to remember procedures. When instructional materials include lengthy explanations or multiple tasks to be performed simultaneously, students' cognitive resources may be devoted to understanding the format of the task rather than the mathematical concept itself. Several improvements are introduced in Cycle II based on the reflection after Cycle I. The student worksheets were simplified by shortening the written instructions and adding more visual illustrations. The teacher also increased the amount of time the children spent exploring, gave immediate feedback, repeated important demonstrations, and used the "point-and-say" technique. This method allowed students to orally state the fraction value of each puzzle piece, write the equivalent mathematical symbol, and point to each puzzle piece. These instructional modifications decreased extraneous cognitive load and supported students' attention to the relationships between the sizes of the pieces, the number of parts, and the fraction symbols.

These changes were found to have a significant effect on the Cycle II results. The average score increased to 78.00 and all five students met the minimum mastery criterion. Student DR increased from a pre-cycle score of 50 to a Cycle II score of 75 and was able to explain that $\frac{1}{2}$ is equivalent to $\frac{2}{4}$. Student IK rose from 45 to 70 and successfully solved adding fractions with like denominators. The highest score in the final score was 85, scored by Student MA, and was able to solve simple word problems independently. The highest improvement was in Student NA, from 40 in pre-cycle to 78 in Cycle II. Meanwhile, student ZAG scored 82 and was able to explain equivalent fractions to peers in the verification stage. The students' progress also indicates the importance of scaffolding. At the start of the intervention, the majority of students were unable to solve fraction tasks on their own. The teacher therefore supported through modeling, guiding questions, verbal prompts, repeating instructions and immediate correction. Support was individualized to each student's needs and slowly phased out as students became more independent. This process relates to Vygotsky's (1978) notion of the Zone of Proximal Development, where learners can accomplish tasks with the right support before they are able to do so on their own.

In particular the progress of students IK and NA demonstrates the importance of structured help. In Cycle I, both students demonstrated hesitance, passivity, and frustration when they could not make sense of the relationship between the puzzle pieces and fraction symbols. In Cycle II, the teacher used shorter instructions, repeated demonstrations and gave immediate feedback when students made errors. Therefore, both students were able to complete the learning tasks successfully and they gained more confidence. This finding suggests that students at risk of being slow learners can learn mathematical concepts when instructional support is matched to their processing speed and learning characteristics. The immediate feedback also had a significant effect on preventing errors from becoming entrenched misconceptions. When students misarranged the puzzle pieces or used the wrong fraction symbol, the teacher immediately asked students to go back and look at the sizes and number of the pieces. Conversely, when students demonstrated correct understanding, the teacher

provided verbal praise and asked students to explain their thinking. Such feedback enabled students to identify their errors, revise their strategies, and strengthen correct conceptual relationships. The intervention also affected students' classroom participation. The students in the pre-cycle were generally passive, hesitated to answer questions and were easily discouraged when dealing with difficult tasks. The results showed that students were more willing to manipulate learning materials, compare the pieces of fractions, ask questions, verify an answer, and communicate their findings after the implementation of Guided Discovery Learning assisted by Fraction Puzzle media. This improvement shows the development of learning engagement as well as conceptual understanding.

The relationship between conceptual understanding and learning participation was reciprocal. The Fraction Puzzle was the first to attract the students' attention and motivated them to participate in the activities. Increased participation then provided more opportunities for students to see, manipulate, compare, and verify fraction relationships. As students started to get the concepts and saw some success, their confidence grew and they became less afraid of making mistakes. The increase in confidence motivated them to participate more actively in the next round of learning activities. The results of this study are in line with the systematic literature review conducted by Annisa, Pandra, and Fauziah (2024) which revealed that Discovery Learning assisted by instructional manipulatives is able to enhance students' conceptual understanding, learning activities, interest, creativity, and mathematics achievement. In both studies the stress is on the importance of combining discovery-oriented learning with concrete materials. However, the present study provides more specific evidence, as it was conducted directly in an inclusive elementary classroom and included students at risk of being slow learners. The result is also supported by Rizkia (2021) who reported that Discovery Learning assisted by circular puzzle media was effective to improve students' understanding of equivalent fractions. The similarity is in the use of "puzzle media" to visualize equivalent fraction relationships. However, this study extends the scope of previous research by considering not only equivalent fractions but also the addition of fractions with common denominators. Additionally, instructional adaptations were implemented in this study, including simplified worksheets, extended processing time, repeated demonstrations, and individualized scaffolding.

In the same way, Alfiani, Fajrie, and Ismaya (2023) found that puzzle media helped improve elementary school students' mathematical conceptual understanding of fractions. Their study showed that the class mean and classical mastery increased from Cycle I to Cycle II. Although the research was carried out through the Course Review Horay model, the results support the use of puzzle media to change the abstract concept of fractions into concrete representations. The present study, however, differed in that it used Guided Discovery Learning and specifically targeted students at risk of being slow learners. The present study therefore contributes to the field of inclusive mathematics education by showing that Discovery Learning can be implemented with students with a slower processing ability of information when the model is transformed into a more structured and guided one. The mathematical content and the learning objectives were not diminished by the instructional adaptations. Instead they changed the way the content was presented with concrete media, simplified language, visual aids, more processing time, practice over and over, immediate feedback, and step-by-step help. These findings indicate that teachers should not automatically lower academic expectations for students at risk of being slow learners. Instead, teachers can keep the intended learning objectives and at the same time vary the instructional procedures according to the cognitive characteristics of students. Students can still learn equivalent fractions and fraction addition, provided the concepts are introduced through meaningful concrete experiences and reinforced through appropriate scaffolding. The findings also suggest that students' errors should be treated as diagnostic information. Misconceptions such as the larger denominator means the larger fraction or adding the numerators and adding the denominators reflect students' interpretation of mathematical symbols. These errors can guide teachers in knowing what sort of concrete experience and guidance is needed. Therefore, corrective instruction should not only give the correct answer, but should help students to reconstruct their understanding through observation and manipulation.

However, this study has certain limitations. The research was limited to a small sample of five students in one elementary school, therefore the generalization of findings is limited. The intervention was also performed for only 2 cycles and concentrated on equivalent fractions and addition of fractions with common denominators. Therefore, the results do not confirm yet whether the understanding of the students would last longer or be transferred to more complex topics such as fractions with different denominators, mixed fractions, decimals or fraction word problems. The overall increase in students' conceptual understanding was not only due to the use of Fraction Puzzle media. It wasn't. It was the combination of concrete manipulation, guided discovery, reduced cognitive load, systematic scaffolding, immediate feedback, extended processing time, and repeated practice. These instructional components were integrated to facilitate students' transition from concrete experiences to symbolic mathematical understanding. Therefore, the modified Guided Discovery Learning model with the assistance of Fraction Puzzle media can be considered as an effective alternative of instruction in improving the conceptual understanding of fractions for students at risk of being slow learners in inclusive elementary classrooms.

CONCLUSION

Based on the adaptive classroom action research conducted in two cycles with five fifth grade students at risk of being slow learners at SDN 009 Kombeng, it can be concluded that the accommodative mathematics instruction on fractions was successfully implemented through a modified Guided Discovery Learning model assisted by Fraction Puzzle manipulative media. The main modifications that contributed to the success of the learning process were the gradual guidance and scaffolding, simplification of instructions in the student worksheets into a more visual form, the extra time for exploration to accommodate slower information-processing speed, and immediate feedback during the exploration and verification stages. This modified instructional model also was shown to improve students' conceptual understanding of equivalent fractions and addition of fractions with common denominators. This improvement was indicated by the increase of the mean score of the conceptual understanding from 48.0 in the pre-cycle with 0% mastery to 64.0 in Cycle I with 60% mastery, and finally to 78.0 in Cycle II with 100% classical mastery, in which all five students had scores above the modified minimum mastery criteria of 65. In light of these results, it is recommended that elementary school teachers, particularly in inclusive classrooms, should avoid over-reliance on lecture-based instruction and abstract symbolic explanations when teaching

students at risk of being slow learners. Rather, teachers should apply modified Guided Discovery Learning and provide concrete manipulative media such as foam or cardboard Fraction Puzzles to decrease students' cognitive load and support enactive learning experiences. Besides, SDN 009 Kombeng is also expected to provide simple but educational manipulative teaching aids in every classroom, carry out teacher training on the characteristics and instructional needs of students who require special educational support, including students who are at risk of being slow learners, to improve inclusive and equitable learning services. For future researchers, it is recommended to study the effectiveness of modified Guided Discovery Learning assisted by manipulative media on more advanced mathematical topics such as operations with fractions with different denominators or decimal numbers as well as to apply the intervention to other groups of students with special educational needs, including students with specific learning disabilities such as dyslexia or dyscalculia considering that this research was limited to only five participants and only covered the basic concepts of fractions.

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