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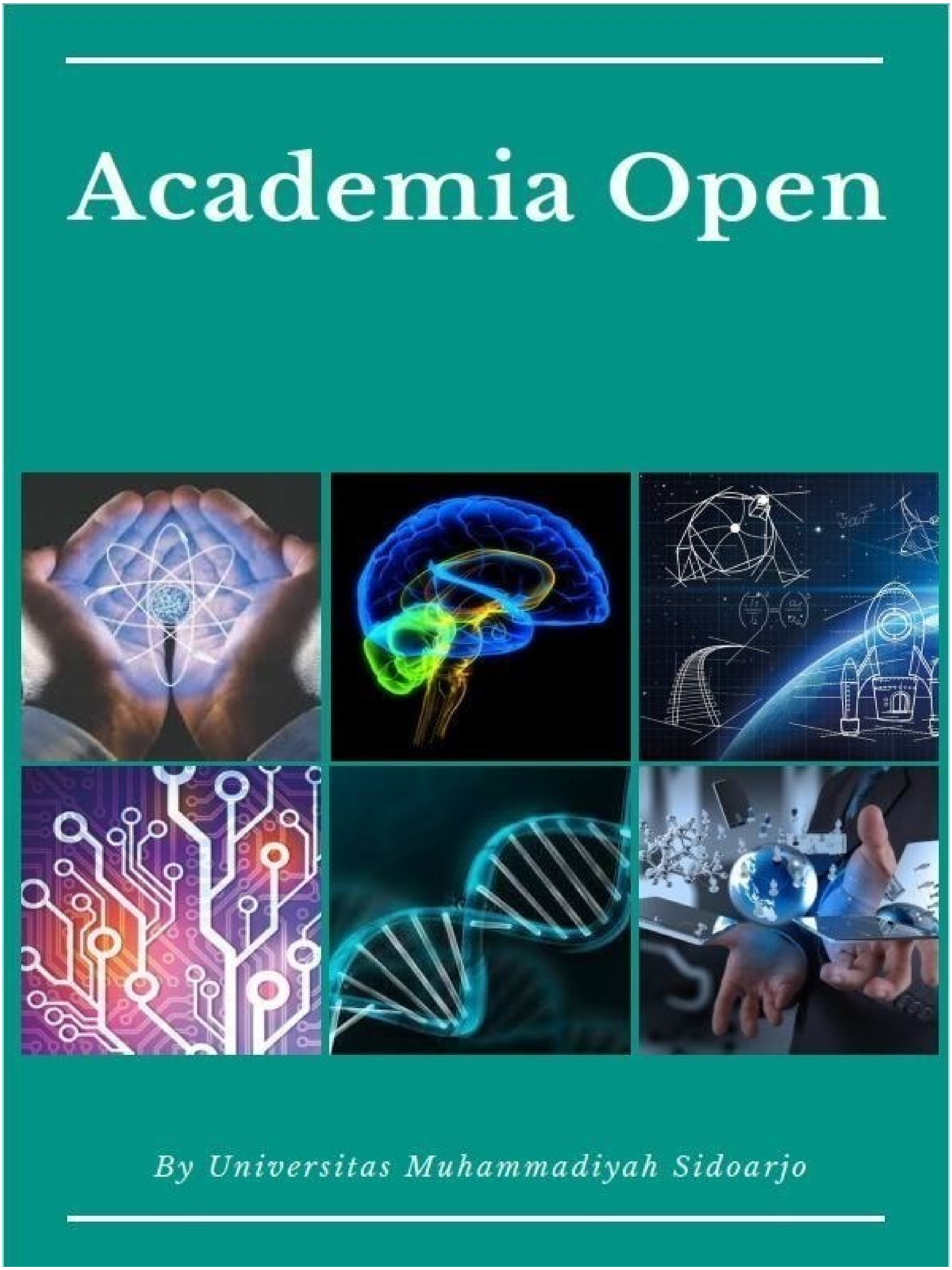

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Method Of Using a Rheological Model in Studying the Process of Compensation of Subgrade Soils with A Vibrating Roller

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Abstract

General Background: Effective soil compaction is fundamental to the stability and durability of road infrastructure, requiring accurate modeling of soil–machine interaction. **Specific Background:** Rheological models have become essential tools for describing elastic, viscous, and plastic behaviors that emerge when vibrating rollers interact with diverse soils, including saline subgrade materials. **Knowledge Gap:** Existing studies have not fully integrated multi-zone rheological behavior into a unified analytical model capable of characterizing deformation dynamics under real vibration loading. **Aims:** This study aims to develop an improved multi-mass rheological model that captures the elastic, viscous, and plastic responses of soils during roller compaction and to derive analytical expressions for predicting deformation characteristics. **Results:** The research presents a three-mass rheological system incorporating Hooke, Newton, and Saint-Venant elements, derives coupled differential equations for soil–roller interaction, and proposes closed-form static and dynamic solutions for displacement and vibration response. The model predicts system stability, resonance susceptibility, damping behavior, and deformation under varying excitation frequencies. **Novelty:** The study integrates soil layer properties into a multi-zone rheological framework, offering a more comprehensive representation of compaction mechanics than prior single-zone or simplified models. **Implications:** The findings provide a scientific basis for optimizing roller parameters, preventing resonance, improving compaction uniformity, and enhancing predictive simulations in road construction engineering.

Highlight :

- The study explains how rheological models represent elastic, viscous, and plastic behavior during roller–soil interaction for more accurate compaction analysis.
- Various models—Hooke, Newton, Maxwell, Kelvin, and Bingham—are applied to describe soil deformation under dynamic loading.
- A three-mass rheological system is developed to predict vibration stability, resonance risk, and soil response during compaction..

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Introduction

In the economy of each state, transport facilities and communication networks (railways, highways and aviation, airfield facilities) are of great importance. Because technological processes at each stage of industry and production are closely related to these areas. In this regard, highways and their infrastructure play a major role in the implementation of logistics services, including the transportation of people, as well as the timely delivery of goods and materials to their intended destination. This, in turn, increases the requirements for road structure elements and increases attention to the quality of technological processes carried out in each layer of soil, as well as the efficiency of the use of machines [1],[2].

The construction of highways is multi-stage, and various machines and mechanisms are used depending on the tasks of the work to be performed.

Including:

- preparatory work (bush cutters, harrows and softeners);

- roadbed construction work (scraper, bulldozer, motor grader, water sprinkler and roller);

- construction of the foundation of the road surface (excavator, bulldozer, motor grader and roller);

- the process of laying road surfaces (truck, asphalt machine, roller);

- surface treatment, drawing road lines and installation of road signs (marking machines, drilling tractor, car crane);

The stages of construction of the road structure are shown in Figure 1.

The compaction process is carried out on all layers of the road structure. Depending on the type of work being performed and the characteristics of the soil being used, compaction machines are used. As we know, the purpose of compacting layer soils is to ensure their strength, load-bearing capacity, and prevent subsidence, displacement, and deformation In this regard, technological processes carried out by compaction machines are important [3]-[6].

Ensuring unquestionable quality indicators requires the research of the interaction processes between the working bodies of the cage and the ground. In this regard, the world's leading scientists, including Akesson F., Beainy F., Darabi M.K., Dongre R., Geske D.M., Huan Q., Kole L.L., Rakowski S., Ryan S., Pellinen T.K., Scherocman J.A., Schwartz C.W, Serafin P.J., West R.C., Witczak, Zubkov A.F., Ivanchenko S.N., Kalujskiy Ya.A., Nosov S.V., Prusov A.Yu., Putka A.I., Repin S.V., Kharkhuta N.Ya., Chabutkin E.K., Shestopalov A.A., Zakharenko A.V., Permyakov V.B., Molokov L.V. Kustarev G.V. Research is being conducted by Uzbek scientists T.K. Khankelov and A.D. Kayumov.

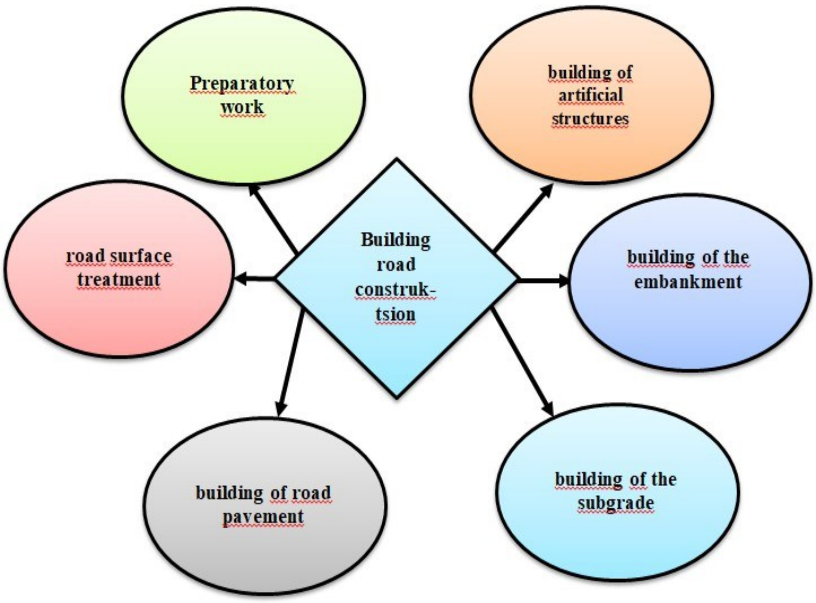

Figure 1. Road structure construction stages

Materials and Methods

In recent years, rheological models have been widely used in studying the physical nature of the interaction between the roller and the external environment, in particular, in studying the compaction process. Through these types of models, the state occurring in each technological process, that is, the mechanical changes in the material under the influence of external forces, are described through rheological model schemes reflecting elastic, plastic, and viscous properties.

In this direction, leading foreign researchers of the world Guiyan Xing, K. Terzaghi, W.A. Lewis, Mooney, Michael A. Robert V. Rinehart, CIS scientists V.I. Balovnev, A.V. Zakharenko, N.N. Ivanov, G.V. Kustarev, Yu.M. Lvovich, V.V. Mikheev, S.V. Nosov, V.B. Permyakov, S.V. Savelyev, N.Ya. Kharkhuta, and Uzbek scientists A.D., Kayumov, Scientists such as T.K. Hankelov have been paying attention to this issue in their research. The types of rheological models and calculation expressions are presented in the studies [7]-[10]. The types of rheological models are selected depending on the nature and operating conditions of the technological process carried out by earthmoving and road construction machines in their interaction with soils. The environment, appearance, calculation expressions and names of the models modeled using these models are presented in Table 1.

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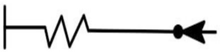
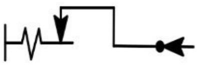
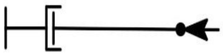
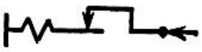
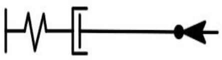
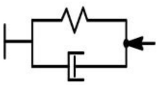
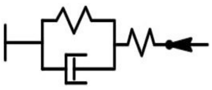
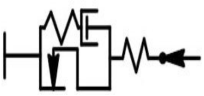
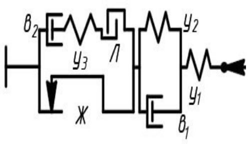
Table 1. The main types of rheological models

Modeled environment	Rheological model		Model name
	Mechanical form	**Mathematical notation**	
Elastic		$\sigma = \varepsilon \cdot E ; \tau = \gamma \cdot G$	Hook
Plastic		$\tau = \tau_{pl} ;$	Saint-Venant
Shoemaker		$\tau = \eta \frac{d\theta}{dz} ;$	Newton
Elasticplastic		$\tau = \tau_{pl} ; \tau = \gamma \cdot G;$	Prandtl
Elastic plastic: Relaxation stress		$\tau = \eta \frac{d\theta}{dz} ; \tau = \gamma \cdot G;$	Maxwell
With delayed deformation		$\tau = \gamma \cdot G + \eta \frac{d\theta}{dz} ;$	Foygt
With the displacement event		$\tau = \gamma \cdot G ;$ $\tau = \gamma_1 \cdot G_1 + \eta \frac{d\theta}{dz} ;$	Kelvin
Elastic-viscous-plastic		$\tau = \gamma \cdot G ;$ $\tau = \gamma_1 \cdot G_1 + \tau_{pl} ;$	Bingam
Elastic viscoplastic relaxation		$\tau = \gamma \cdot G ;$ $\tau = \gamma_1 \cdot G_1 + \tau_{pl} ;$ $\tau = \tau_{pl} + \eta \frac{d\theta}{dz}$	Shvedov
Soil in the process of shear deformation		$\tau = \gamma \cdot G ;$ $\tau = \gamma_1 \cdot G_1 + \eta \frac{d\theta}{dz} ;$ $\tau = \tau_{pl} + \eta_1 \frac{d\theta}{dz} .$	Combined

The above models are general rheological models, and they come in different versions depending on the types of compaction machines used, the shape and form of the working body, as well as the physical and mechanical properties of the interacting layer soils. Models of this type have been

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cited in research works. In particular, in the research of foreign scientists Michael A. Mooney and Robert V. Rinehart [11], field studies were conducted using a vibrating roller compactor equipped with devices to study the relationship between vibration characteristics and the properties of the lower part of the soil, namely soil porosity. Devices were installed on the roller to control the acceleration of the drum and frame, as well as the force of the eccentric.

It was found that both the acceleration and the phase of rotation of the drum are very sensitive to changes in the base soil. Considering only vertical displacement, the model in Figure 2a includes the masses of the drum and frame, respectively, and the spring and the cover, which represent the rubber vibration-isolating support of the drum and frame. The time-varying force transmitted to the ground can be easily determined using the force balance (Figure 2b).

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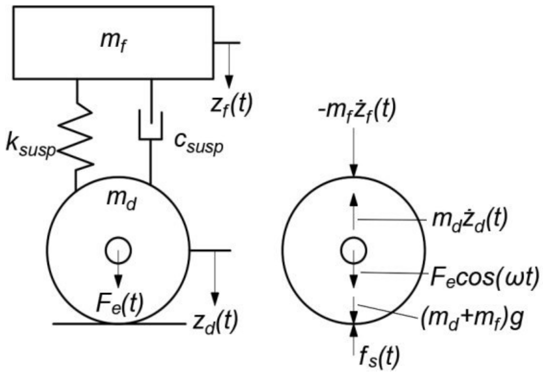

Figure 2. (a)Two-DOF, lumped mass parameter representation of roller compactor; (b) free body diagram of drum atop soil

$$f_s(t) = F_e \cos(\omega t) + (m_d + m_f) \cdot - m_d \cdot \ddot{z}_d - m_f \ddot{z}_f \quad (1)$$

where F_e - vertical force amplitude due to the rotating eccentric $m_0 e_0 \omega^2$; ω -eccentric excitation frequency rad/s; g - acceleration due to gravity; $\ddot{z}_d$ and $\ddot{z}_f$ acceleration of the drum and frame respectively.

In the studies of S.V Savelev and V.V Mikheev, when studying the deformation stress state of the medium in the state of compaction, the interaction of the external load was considered as a conditionally elastic rod, and the contact surface with the compacted medium was taken as a marked dimension S, Figure 3 [12].

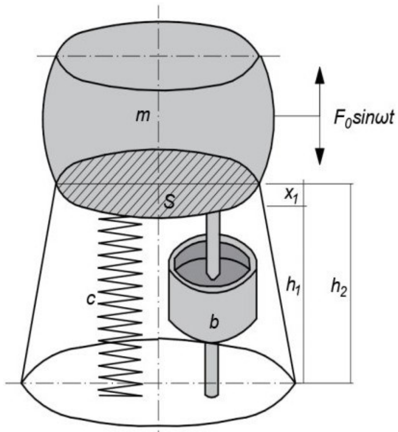

Figure 3. Diagram of the interaction of an elastically deformable rod with an external load through the contact surface of the stamp

Based on this model, we write the effect of dynamic force on the deformable elastic viscous medium in the form of a differential equation.

$$m\Delta\ddot{x} + 2b\hat{x} + c\Delta x = F_0 \sin \omega t \quad (2)$$

where m - the mass of the soil actively “connected” with the working body of the compaction, kg; Δx - soil deformation during one cycle of loading, m; b - deformable soil viscosity, n/m²; c - elasticity of the deformable “column” environment, n/m; - vibration excitation force, N; ω - frequency, s⁻¹; S-contact area, m².

In turn, the elasticity of the deformable “column” environment is determined by this formula.

$$c = \frac{E \cdot S}{h} \quad (3)$$

where is the modulus of elasticity of the deformable medium, Pa.

Also, the deformable soil viscosity is determined from this expression:

$$\eta \cdot S$$

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$$b = \frac{}{2 \cdot h} \tag{4}$$

Then expression (1) becomes:

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$$m\Delta\ddot{x} + 2\frac{\eta \cdot S}{2 \cdot h}\hat{x} + \frac{E \cdot S}{h}\Delta x = F_0 \sin \omega t \quad (5)$$

If we divide the left and right sides of the equation by the mass, then the following formula is formed:

$$\Delta\ddot{x} + \frac{\eta \cdot S}{m \cdot h}\hat{x} + \frac{E \cdot S}{m \cdot h}\Delta x = \frac{F_0 \sin \omega t}{m} \quad (6)$$

AA. Beloded's research focused on the problems of the mathematical model of the dynamic deformation process of the compressing elastic viscous plastic medium, and Hook, Newton and Saint-Venant elements were selected for this process [13]. Taking into account many assumptions, an elemental column of a dense medium with a definite mass and volume, acting on an external periodic force, was seen (Fig. 4).

We express the influence of the external periodic force of the frequency vibrator on the deformable soil in the form of the differential equation of the movement of the mass of the environment as follows.

$$\rho \cdot V\Delta\ddot{x} + b_2\dot{x} + c_2\Delta x = F_0 \sin \omega t + F_{cm} \quad (7)$$

where Δx - deformation of the medium, m: $m = \rho \cdot V$ - mass of the medium, kg, ρ - density of the deforming medium, kg/m3; V -volume of the deforming column of the medium, m3; $F_{cm} = mg$ - weight of the working body, N; c_2 - stiffness of the deforming column of the medium, N/m; b_2 - coefficient of viscous friction of the deforming volume.

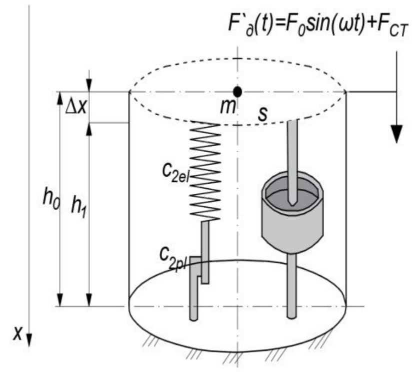

Figure 4. Scheme of deformation of a column of elementary elastic plastic viscous deformable medium under the influence of external force

Researcher E.A. Shishkin in his scientific research determined the parameters of the rheological model based on laboratory studies [14]. The model proposed by the scientist shows the elastic (c_1), viscous-elastic (c_2, μ_2) properties of the material (Fig. 5).

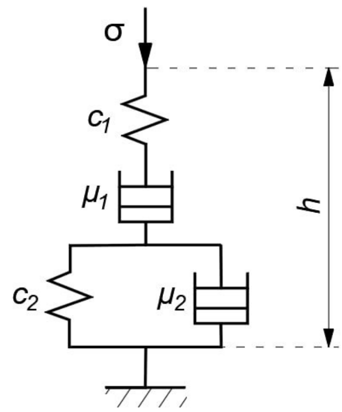

Figure 5. Model of asphalt concrete mixture

The above properties are determined based on the results of laboratory tests on the ductility-recovery of the material. The model in Figure 5 is a special case of the generalized Kelvin model, for which the relationship between stress and deformation is as follows:

$$h = \frac{\sigma}{c_1} + \frac{\sigma}{\mu_1 \cdot \delta_1} + \frac{\sigma}{c_2 + \mu_2 \cdot \delta_1} \quad (8)$$

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Where h - deformation, m; σ -tension, Pa; , c_1, c_2 -unity coefficients, N/m³; , μ_1, μ_2 -viscosity coefficients, Ns/m³; δ_1 -linear time differentiation operator. By doing what is shown in expression (6), we get the defining equation for the model shown in Figure 5.

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$$\frac{\mu_2}{c_1}\sigma^{//} + \left(1 + \frac{c_2}{c_1} + \frac{\mu_2}{\mu_1}\right)\sigma^{/} + \frac{c_2}{\mu_1}\sigma = \mu_2 h^{//} + c_2 h^{/} \quad (9)$$

Results and Discussion

Continuing their research in the field of rheological models, the model proposed by them was improved [15]. The state of the interaction between the working body of the compaction machine and the soil is divided into 3 parts. Zone 1 is the elastic , zone 2 is viscous zone, and zone plastic, 3 . The compacted soil layer is also taken into account in the formulas describing the process. The model developed based on the above ideas is presented in Figure 6 in the frame-roller-soil system.

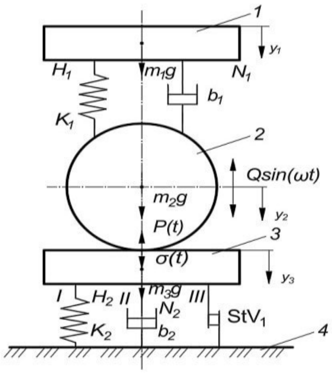

Figure 6. A three-mass rheological model system

here is the loading of frame 1; 2 - rollers; 3 - road base soil; 4 - base; N1, N2 - Newton's models; H1, H2 - Hooke's models; StV - Sen Venan model.

Based on this scheme, the following system of expressions can be given.

$$m_1 \ddot{y}_1 - b_1 \cdot (\dot{y}_1 - \dot{y}_2) - k_1(y_1 - y_2) = m_1 g \quad (10)$$

$$(m_2 + m_3) \cdot \ddot{y}_2 + b_2 \cdot \dot{y}_2 + b_1(\dot{y}_2 - \dot{y}_1) + \kappa_2 \cdot y_2 + k_1(y_3 - y_1) = (m_2 + m_1) \cdot g + P(t); \quad (11)$$

$$\dot{y}_2 = \dot{y}_3;\ y_2 = y_3;\ StV_1 = 0. \quad (12)$$

where m_1 - load mass (valets frame mass), kg; m_2 - mass of valets transmitting harmonic vibrations from the vibrator, kg; y_1 -amplitude of coil body vibration generated by the rotation of the valets imbalance shaft, mm; y_2 -valets vibration amplitude, mm; y_3 - displacement created in the soil; b_1 - coefficient of relative viscosity in the combination of valets and frame, $N \cdot s / m$; b_2 - relative viscosity coefficient of soil, $N \cdot s / m$; k_1 - relative resistance coefficients of valets and frame connecting dampers, N/m; k_2 - elasticity resistance coefficient of soil N/m; $P(t)$ - excitation power of the vibrator, N; ω - angular frequency of rotation of the vibrator shaft, rad/s; t - oscillation time, s; $\sigma(t)$ - the impact reaction of compacted road base soil on valets, N.

If we introduce the following conditions:

$$\dot{y}_1 - \dot{y}_2 = \Delta\dot{y}_{1,2};\ y_1 - y_2 = \Delta y_{1,2};\ y_3 - y_1 = \Delta y_{1,3};\ \dot{y}_2 - \dot{y}_1 = -\Delta\dot{y}_{1,2} \quad (13)$$

Then expressions (10) and (11) take the following form.

$$m_1 \ddot{y}_1 - b_1 \Delta\dot{y}_{1,2} - k_1 \Delta y_{1,2} = m_1 g$$

$$(m_2 + m_3) \cdot \ddot{y}_2 + b_2 \cdot \dot{y}_2 - b_1 \Delta\dot{y}_{1,2} + \kappa_2 \cdot y_2 + k_1 \Delta y_{1,3} = (m_2 + m_1) \cdot g + P(t)$$

$$\dot{y}_2 = \dot{y}_3;\ y_2 = y_3$$

The system of equations (14 and 15) can be written as follows:

Based on this rheological model, the equations can be expressed in this form:

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(15)

(14)

(16)

$$M \cdot \ddot{y} + C\dot{y} + ky = P(t) \quad (17)$$

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We write the equation (17) in matrix form.

$$M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 + m_3 \end{pmatrix}; B = \begin{pmatrix} b_1 & -b_1 \\ -b_1 & b_1 + b_2 \end{pmatrix}; k = \begin{pmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{pmatrix} \quad (18)$$

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}; \; F(t) = \begin{pmatrix} m_1 g \\ (m_2 + m_3)g + P(t) \end{pmatrix} \quad (19)$$

$$y_2 = y_3 \Rightarrow k_1(y_3 - y_1) = k_1(y_2 - y_1) \quad (20)$$

We solve the static part of the equation

$$P(t) = 0 \quad (21)$$

In the static field

$$\dot{y} = 0; \; \ddot{y} = 0 \quad (22)$$

$$-k_2(y_{1c} - y_{2c}) = m_1 g \Rightarrow y_{1c} = y_{2c} - \frac{m_1 g}{k_1} \quad (23)$$

$$k_2 y_{2c} + k_1(y_{2c} - y_{1c}) = (m_1 + m_2)g \quad (24)$$

Substituting expression (23) into expression (24), we calculate:

$$k_2 y_{2c} + k_1(y_{2c} - (y_{2c} - \frac{m_1 g}{k_1}) = (m_1 + m_2)g \quad (25)$$

If we simplify the equation (25), the following expression is formed:

$$y_{2c} = \frac{m_2 g}{k_2} \quad (26)$$

Expressions (23) and (26) are solutions of equations (10, 11, 12).

Equations (10, 11, 12) can be given in the following form:

$$y_{1c} = \frac{m_2 g}{k_2} - \frac{m_1 g}{k_1} \quad (27)$$

To calculate the dynamic part, we introduce the following relative coordinates:

$$y_1(t) = y_{1c} + z_{1(t)}, \; y_2(t) = y_{2c} + z_{2(t)} \quad (28)$$

Since gravity forces are calculated in the static solution, they do not participate in the following equation.

$$m_1 \ddot{z}_1 - b_1 \cdot (\dot{z}_1 - \dot{z}_2) - k_1(z_1 - z_2) = m_1 g; \quad (29)$$

$$(m_2 + m_3) \cdot \ddot{z}_2 + b_2 \cdot \dot{z}_2 + b_1(\dot{z}_2 - \dot{z}_1) + \kappa_2 \cdot z_2 + k_1(z_3 - z_1) = P(t); \quad (30)$$

$$M \cdot \ddot{z} + B\dot{z} + kz = P(t), \quad (31)$$

$$\begin{pmatrix} m_1 & 0 \end{pmatrix} \quad \begin{pmatrix} b_1 & -b_1 \end{pmatrix} \quad \begin{pmatrix} k_1 & -k_1 \end{pmatrix}$$

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$$M = \begin{pmatrix} 0 & m_2 + m_3 \end{pmatrix}, \quad B = \begin{pmatrix} -b_1 & b_1 + b_2 \end{pmatrix}; \quad k = \begin{pmatrix} -k_1 & k_1 + k_2 \end{pmatrix}, \tag{32}$$

$$y = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}; \quad F(t) = \begin{pmatrix} 0 \\ P(t) \end{pmatrix}. \tag{33}$$

The solution of the differential equation (31) consists of the sum of 2 solutions.

$$z(t) = z_u + z_x \tag{34}$$

Where z_u - the general solution of the homogeneous part; z_x - (31) is a particular solution of the equation.

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$$M\cdot\ddot{z}_u + B\dot{z}_u + kz_u = 0 \tag{35}$$

$$z_u(t) = C_1\cos(\omega_1 t + \varphi_1) + C_1\sin(\omega_2 t + \varphi_2) \tag{36}$$

The specific solution of the equation (31) is selected depending on the form of power transmission.

For example, if it is in the form, then we look for a specific solution of equation (31) in the following form:

$$z_x = A_1\sin(\omega_3 t) + B_1\cos(\omega_3 t) \tag{37}$$

$$\dot{z}_x = A_1\omega_3\cos\omega_3 t + B_1\omega_3\sin\omega_3 t \tag{38}$$

$$\ddot{z}_x = A_1\omega_3^2\sin t + B_1\omega_3^2\cos t \tag{39}$$

$$M\cdot\ddot{z}_x + B\dot{z}_x + kz_x = P_0\cdot\sin\omega_3 t \tag{40}$$

$$A_1(k - M\omega_3^2) - B\omega_3 B_1 = P_0 \tag{41}$$

$$A_1(k - B\omega_3) - M\omega_3^2 B_1 = 0 \Rightarrow B_1 = \frac{k - B\omega_3}{M\omega_3^2}A_1 \tag{42}$$

$$A_1\left(k - M\omega_3^2 - \frac{k - B\omega_3}{M\omega_3^2}B\omega_3\right) = P_0 \tag{43}$$

$$A_1((k - M\omega_3^2)\cdot M\omega_3 - B(k - B\omega_3)) = P_0\cdot M\omega_3 \tag{44}$$

If we introduce the following definition

$$(k - M\omega_3^2)\cdot M\omega_3 - B(k - B\omega_3) = C$$

in that case

$$A_1 = \frac{P_0\cdot M\omega_3}{C} \tag{45}$$

$$B_1 = \frac{k - B\omega_3}{C\omega_3}\cdot P_0 M \tag{46}$$

$$z(t) = z_u + z_x = C_1\cos(\omega_1 t + \varphi_1) + C_1\sin(\omega_2 t + \varphi_2) + A_1\sin(\omega_3 t) + B_1\cos(\omega_3 t) \tag{47}$$

$$y(t) = y_c + z(t) \tag{48}$$

$$y(t) = \frac{m_2 g}{k_2} - \frac{m_1 g}{k_1} + C_1\cos(\omega_1 t + \varphi_1) + \tag{49}$$

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$$C_1 \sin(\omega_2 t + \varphi_2) + + A_1 \sin(\omega_3 t) + B_1 \cos(\omega_3 t)$$

This general solution (49) helps to predict the stability of a mechanical system, its susceptibility to resonance, amplitude increases under load, and damping characteristics. In the future, this formula will serve as a basic tool in the following scientific and practical areas:

1. selection of optimal design parameters to reduce vibrations;
2. determination of operating frequencies to avoid resonance;
3. estimation of damping coefficients in the design of active or passive shock absorbers;
4. prediction of dangerous vibrations in machines and units;
5. more accurate construction of modeling and numerical simulations of mechanical systems.

Thus, this formula provides the necessary scientific and methodological basis for a deep analysis of the dynamic characteristics of the system, optimization of the design, and prediction of future vibration problems.

Conclusion and Recommendations

The above differential formulas are analytically difficult to calculate. Therefore, software tools are used to obtain the resulting values. Ongoing scientific research shows that rheological models characterizing the interaction between the compactor and the soil during compaction of road structure soils fully reflect the state of deformation in them. In turn, the soils, mixtures and construction materials used in each layer of soil are distinguished by their own characteristics and properties.

In particular, the main soil parameters for road base construction are optimal moisture content and maximum density, which are closely related to the amount of water in the soil, in particular, the wetting process. In asphalt mixtures, viscosity is one of the main characteristics, which directly affects the quality of bitumen and mineral powder, and the technology of preparation of the mixture in the final state is of great importance

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