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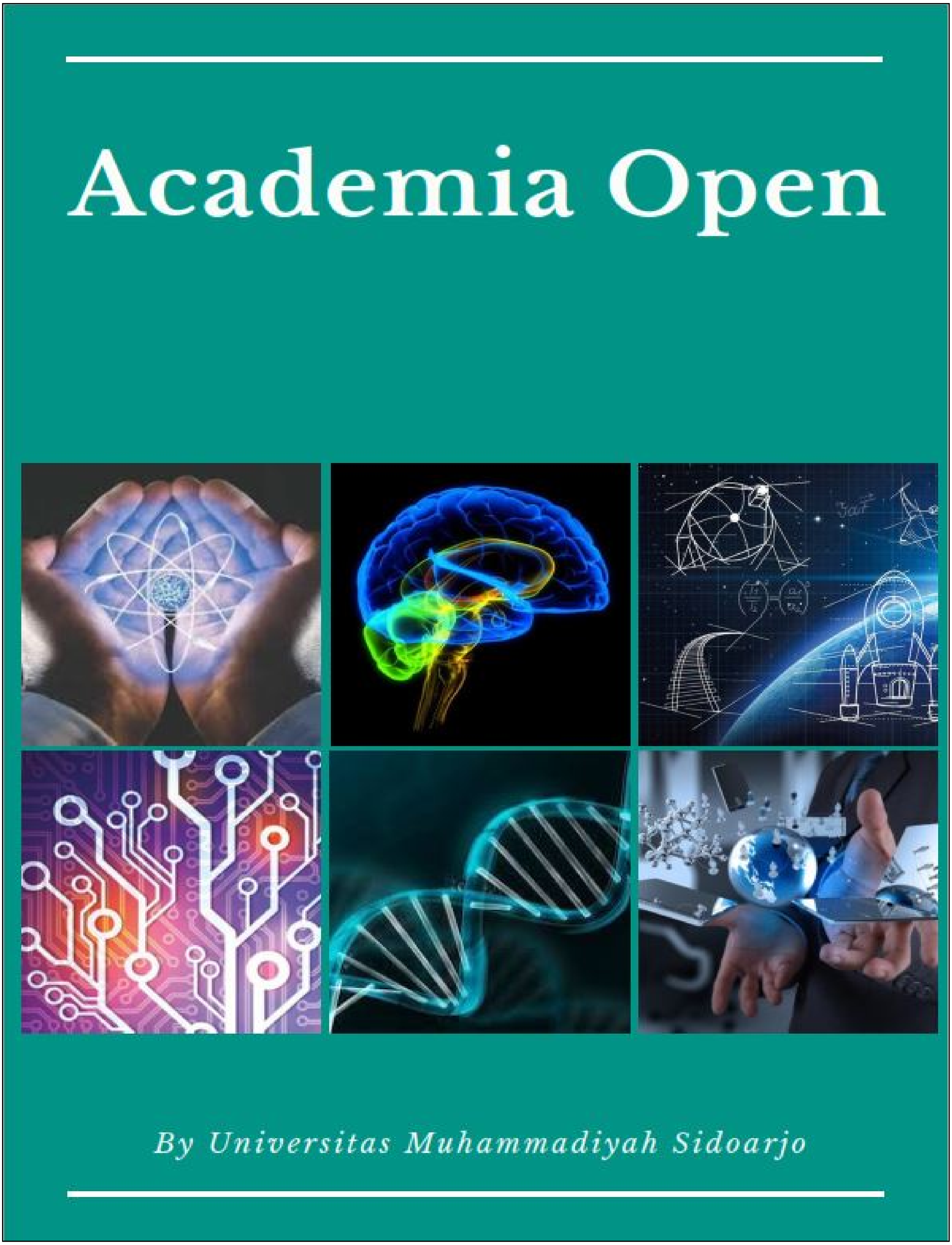

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Comparing Different Fuzzy Estimator of Hazard Rate for Quasi Lindely Distribution

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Abstract

General Background: Fuzzy estimation plays a vital role in enhancing the precision of statistical inference under uncertainty, particularly in reliability theory. **Specific Background**: Classical estimators often struggle with mixed probability distributions involving both continuous and vague components. **Knowledge Gap**: Despite the theoretical relevance, limited comparative analysis exists on fuzzy estimators within hybrid exponential-gamma models under varied risk functions. **Aim**: This study aims to derive and compare various fuzzy estimators for the risk function of a mixed continuous distribution formed by combining the exponential (θ) and Gamma (2,θ) distributions, with mixing proportions β/(β+1) and 1/(β+1), respectively. **Results**: We derive the corresponding probability density function (pdf), cumulative distribution function (CDF), reliability, and hazard functions. A fuzzy vagueness factor (k̃) is introduced into the hazard equation, and the r-th raw moment [μ′(r)] is formulated. Parameters θ and β are estimated via maximum likelihood, moments, and frequency ratio methods. **Novelty**: The integration of fuzzy theory into hazard modeling for a quasi-Lindley framework, coupled with comprehensive estimator comparison, offers novel insights. **Implications**: The findings enhance reliability analysis under fuzzy environments, enabling more robust decision-making in engineering and survival analysis contexts.

Highlights:

- Introduces fuzzy estimation in hazard function modeling.
- Compares three estimation methods for mixed distributions.
- Derives complete reliability metrics from a hybrid model.

Keywords: Maximum Likelihood, Moment Estimation, Fuzzy Estimator, Hazard Rate, Quasi-Lindley Distribution

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Introduction

The two parameters Quasi – Lindely (QLD) distribution is one of continuous probability distribution constructed from eponential with parameter (θ) a to parameters Gamma $2,\theta)$, with mixing proportion$\left(\frac{\beta}{\beta+1},\frac{1}{\beta+1}\right)$. This distribtion was studied and introduced by different researchers like, Shanker and Mishra [1], This distribution (QLD) can be used for modelling, Sankaran M. [2], who introduce discrete poisson Lindely. also Zakerzada H. Dolati A. [3] work on generalization of quasi lindely. Ghitany M.E. Atieh, B. and Nadarajah S. [4] made many applications of quasi lindely in mathematics and computing simulation. Here we do not need to enumerate all the studies of this subject, but we would like to indicate into extension of quasi lindely with another distribution like quasi Poisson Lindely as introduced by Shanker and Mishra [5], we continue the work about this distribution and comparing different estimators of fuzzy hazard rate function using simulation [6].

1. Theoretical Aspect

Let;

$$f_1(x) = \theta e^{-\theta x} \quad (1)$$

$$f_2(x) = \theta^2 x\, e^{-\theta x} \quad (2)$$

$$f(x,\theta,k) = k f_1(x) + (1-k)\, f_2(x) \quad (3)$$

Where;

$$k = \frac{\beta}{\beta+1}$$

Then;

$$f(x,\theta,\beta) = \frac{\beta}{\beta+1}\theta e^{-\theta x} + \frac{1}{\beta+1}\,\theta^2 x\, e^{-\theta x} \qquad x,\theta > 0, \beta > -1 \quad (4)$$

Equation (4) can be written as;

$$f(x,\theta,\beta) = \frac{\beta\theta e^{-\theta x} + \theta^2 x\, e^{-\theta x}}{\beta+1} = \frac{\theta(\beta+\theta x)e^{-\theta x}}{\beta+1} \quad (5)$$

The cumulative distribution function (CDF) is; [7]

$$F_X(x) = \int_0^x f(u)du = \int_0^x \frac{\theta(\beta+\theta u)e^{-\theta u}}{\beta+1}du \quad (6)$$

$$F(x) = 1 - \frac{(1+\beta+\theta x)e^{-\theta x}}{\beta+1} \quad (7)$$

The reliability function; [8]

$$R_X(x) = 1 - F_X(x) = \frac{(1+\beta+\theta x)e^{-\theta x}}{\beta+1} \quad (8)$$

While the hazard rate function;

$$h(x) = \frac{f(x)}{R_X(x)} = \frac{\theta(\beta+\theta x)}{(1+\beta+\theta x)} \quad (9)$$

when $x_1 < x_2 \Longrightarrow h(x_1) < h(x_2)$, and $h(x)$ for quasi Lindely represent monotone increasing function.

We can prove that the r^{th} formula for the moments about origin is; [9]

$$\mu_r' = E(x^r) = \int_0^\infty x^r\; f(x,\beta,\theta)dx$$

This gives; [10]

$$E(x^r) = \frac{\beta\Gamma(r+1) + (r+1)\Gamma(r+1)}{(\beta+1)\theta^r} \quad (10)$$

Then; [11] [12] [13]

$$E(x) = \frac{\beta\Gamma(2)+2\Gamma(2)}{(\beta+1)\theta} = \Gamma(2)\frac{(\beta+2)}{(\beta+1)\theta} = \frac{(\beta+2)}{(\beta+1)\theta} \quad (11)$$

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$$E(x^2) = \frac{\beta\Gamma(3)+3\Gamma(3)}{(\beta+1)\theta^2} = \frac{2\beta+6}{(\beta+1)\theta^2} \quad (12)$$

$$v(x) = \frac{\beta^2+4\beta+2}{(\beta+1)\theta^2} \quad (13)$$

Method

A. Estimation of Parameters

We present different methods to estimate (θ, β), and to estimate fuzzy hazard function $[\tilde{h}(x)]$.

1. Maximum Likelihood Method

Let $(x_1, x_2, \dots, x_n)$ be a random variable from p.d.f in equation (5), then; [14] [15] [16]

$$L = \prod_{i=1}^{n} f(x_i, \beta, \theta) = \theta^n (\beta+1)^{-n} \prod_{i=1}^{n} (\beta+\theta x_i)\, e^{-\theta \sum_{i=1}^{n} x_i} \quad (14)$$

$$\log L = n \log\theta - n\log(\beta+1) + \sum_{i=1}^{n} \log(\beta+\theta x_i) - \theta \sum_{i=1}^{n} x_i$$

$$\frac{\partial \log L}{\partial \theta} = \frac{n}{\theta} + \sum_{i=1}^{n} \frac{x_i}{(\beta+\theta x_i)} - \sum_{i=1}^{n} x_i = 0$$

$$\hat{\theta}_{MLE} = \frac{n}{\left(\sum_{i=1}^{n} \frac{x_i}{(\beta+\theta x_i)} - \sum_{i=1}^{n} x_i\right)} \quad (15)$$

$$\frac{\partial \log L}{\partial \beta} = -\frac{n}{(\beta+1)} + \sum_{i=1}^{n} \frac{1}{(\beta+\theta x_i)} = 0$$

$$(\hat{\beta}+1) = \frac{n}{\left(\sum_{i=1}^{n} \frac{1}{(\hat{\beta}+\hat{\theta} x_i)}\right)}$$

$$\hat{\beta}_{MLE} = \frac{n}{\left(\sum_{i=1}^{n} \frac{1}{(\hat{\beta}+\hat{\theta} x_i)}\right)} - 1 \quad (16)$$

2. Method of Moments

From equations (11 & 12), then from equating; [17]

$$E(x) = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{(\hat{\beta}+2)}{(\hat{\beta}+1)\theta} = \bar{x} \Longrightarrow (\hat{\beta}+1)\theta = \left(\frac{(\hat{\beta}+2)}{\bar{x}}\right)$$

$$E(x^2) = \frac{\sum_{i=1}^{n} x_i^2}{n} = \frac{2\beta+6}{(\beta+1)\theta^2}$$

$$\hat{\theta}_{MOM} = \frac{(\hat{\beta}_{MOM}+2)}{(\hat{\beta}_{MOM}+1)\,\bar{x}} \quad (17)$$

We can obtain $(\hat{\beta}_{MOM})$ as; [18]

$$M_2 \hat{\theta}_{MOM}^2 (\hat{\beta}_{MOM}+1) - 2(\hat{\beta}_{MOM}+3) = 0 \quad (18)$$

Where $M_2 = \left(\frac{\sum_{i=1}^{n} x_i^2}{n}\right)$

3. Estimation by Frequency Ratio

Let; [19]

$$f(x_1, \beta, \theta) = \frac{\beta\theta}{\beta+1} e^{-\theta x_1} + \frac{\theta^2 x_1}{\beta+1} e^{-\theta x_1} = \frac{\beta\theta}{\beta+1} e^{-\theta x_1} (1+\theta x_1)$$

$$f(x_2, \beta, \theta) = \frac{\beta\theta}{\beta+1} e^{-\theta x_2} + \frac{\theta^2 x_2}{\beta+1} e^{-\theta x_2} = \frac{\beta\theta}{\beta+1} e^{-\theta x_2} (1+\theta x_2)$$

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$$\frac{f(x_1,\beta,\theta)}{f(x_2,\beta,\theta)} = \left(\frac{e^{-\theta x_1}(1+\theta x_1)}{e^{-\theta x_2}(1+\theta x_2)}\right)$$

$$\ln\frac{f(x_1,\beta,\theta)}{f(x_2,\beta,\theta)} = \ln\left(\frac{e^{-\theta x_1}(1+\theta x_1)}{e^{-\theta x_2}(1+\theta x_2)}\right)$$

$$\ln\frac{f(x_1,\beta,\theta)}{f(x_2,\beta,\theta)} = -\,\theta(x_1 - x_2) + \ln(1+\theta x_2) - \ln(1+\theta x_1)$$

$$= -\theta(x_1 - x_2) + \ln\left(\frac{1+\theta x_1}{1+\theta x_2}\right)$$

$$\hat{\theta}(x_1 - x_2) = \ln\left(\frac{1+\theta x_1}{1+\theta x_2}\right) - \ln\frac{f(x_1,\beta,\theta)}{f(x_2,\beta,\theta)}$$

$$\hat{\theta}(x_1 - x_2) = \ln\left(\frac{[1+\theta x_1]f(x_1,\beta,\theta)}{[1+\theta x_2]f(x_2,\beta,\theta)}\right)$$

$$\hat{\theta}_{FRE} = \ln\left[\frac{[1+\theta x_1]f_1}{[1+\theta x_2]f_2} \times \frac{1}{(x_1-x_2)}\right] \quad (19)$$

Results and Discussion

A. Simulation

We introduce the results of simulation for $[\tilde{h}(x)]$ after we estimate the two parameters (θ, β) by methods of moments, maximum likelihood, and method of frequency ratio.

$$\tilde{h}(x_i) = \frac{f(x_i)}{R_X(x_i)} = \frac{\hat{\theta}(\hat{\beta}+\hat{\theta}\tilde{k}x_i)}{(1+\hat{\beta}+\hat{\theta}\tilde{k}x_i)}$$

Taking $(n = 20,40,60,80)$

$\boldsymbol{\beta}$	0.7	1.5	
$\boldsymbol{\tilde{k}}$	0.2	0.5	0.8
$\boldsymbol{\theta}$	0.5	0.8	

Table 1. Estimated values of (β,θ,k̃).

$\boldsymbol{n}$	$\boldsymbol{x_i}$	$\boldsymbol{\tilde{h}(x_i)}$	MOM $\boldsymbol{\tilde{h}(x_i)}$	MLE $\boldsymbol{\tilde{h}(x_i)}$	FR $\boldsymbol{\tilde{h}(x_i)}$	Best
20	2	0.3202	0.3125	0.3962	0.2887	FR
	2.5	0.3763	0.3607	0.3384	0.3282	FR
	3	0.4266	0.3796	0.3762	0.3463	FR
	3.5	0.4452	0.4109	0.3650	0.3667	MLE
	4	0.4826	0.4226	0.4262	0.3824	FR
	4.5	0.5036	0.4465	0.4039	0.3656	FR
40	2	0.3202	0.3523	0.3182	0.3906	MLE
	2.5	0.3763	0.3872	0.3162	0.4132	MLE
	3	0.4266	0.4107	0.4145	0.4022	MOM
	3.5	0.4452	0.4073	0.4256	0.4136	MOM
	4	0.4826	0.4721	0.4362	0.4025	FR
	4.5	0.5036	0.4526	0.4401	0.4016	FR
	2	0.3202	0.3396	0.3378	0.3369	FR
	2.5	0.3763	0.3984	0.3947	0.3987	MLE

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60	3	0.4266	0.4346	0.3449	0.3938	MLE
	3.5	0.4452	0.4611	0.4803	0.3919	FR
	4	0.4826	0.4822	0.4962	0.4972	MOM
	4.5	0.5036	0.5006	0.4993	0.4665	FR
80	2	0.3202	0.4163	0.5066	0.4882	MOM
	2.5	0.3763	0.3812	0.5241	0.4892	MOM
	3	0.4266	0.5206	0.5003	0.4662	FR
	3.5	0.4452	0.4535	0.4426	0.4701	MOM
	4	0.4826	0.4706	0.4632	0.4662	MLE
	4.5	0.5036	0.4821	0.4601	0.4552	FR

Table 2. values of $\tilde{h}(x)$ when (β=0.7,θ=0.5,$\tilde{k}$=0.2,R=1000)

n	x_i	$\tilde{h}(x_i)$	MOM $\tilde{h}(x_i)$	MLE $\tilde{h}(x_i)$	FR $\tilde{h}(x_i)$	Best
20	2	0.2787	0.3124	0.2996	0.2498	FR
	2.5	0.3282	0.3362	0.3031	0.3367	MLE
	3	0.3465	0.3351	0.3542	0.3671	MLE
	3.5	0.3667	0.4042	0.4041	0.4005	FR
	4	0.3824	0.4242	0.4265	0.4209	FR
	4.5	0.4041	0.4061	0.4311	0.4277	MOM
40	2	0.2787	0.2766	0.2786	0.4441	MOM
	2.5	0.3282	0.3092	0.3229	0.3526	MOM
	3	0.3465	0.3352	0.3506	0.3321	FR
	3.5	0.3667	0.3452	0.3916	0.3562	MOM
	4	0.3824	0.3877	0.4006	0.3772	FR
	4.5	0.4041	0.3897	0.4010	0.4152	MLE
60	2	0.2787	0.2667	0.2778	0.2889	MOM
	2.5	0.3282	0.3521	0.3506	0.2997	FR
	3	0.3465	0.4019	0.3302	0.2767	FR
	3.5	0.3667	0.4115	0.3551	0.3441	FR
	4	0.3824	0.3991	0.3721	0.3351	FR
	4.5	0.4041	0.4124	0.3779	0.3656	FR
80	2	0.2787	0.6053	0.5728	0.5703	MLE
	2.5	0.3282	0.6518	0.6216	0.6197	FR
	3	0.3465	0.6802	0.6541	0.6425	FR
	3.5	0.3667	0.7032	0.6773	0.6525	FR

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	4	0.3824	0.7021	0.7081	0.6643	FR
	4.5	0.4041	0.7066	0.7189	0.6682	FR

Table 3. values of $\tilde{h}(x)$ when (β=0.7,θ=0.8,$\tilde{k}$=0.2,R=1000)

n	x_i	$\tilde{h}(x_i)$	MOM $\tilde{h}(x_i)$	MLE $\tilde{h}(x_i)$	FR $\tilde{h}(x_i)$	Best
20	2	0.6620	0.6064	0.6544	0.5627	FR
	2.5	0.6720	0.6518	0.6132	0.6331	MLE
	3	0.6730	0.6880	0.6452	0.6522	MLE
	3.5	0.7020	0.7032	0.7072	0.6864	FR
	4	0.7750	0.7192	0.7188	0.6882	MLE
	4.5	0.7781	0.7236	0.7321	0.6992	FR
40	2	0.6620	0.6452	0.6504	0.5582	FR
	2.5	0.6720	0.6230	0.6040	0.6092	FR
	3	0.6730	0.6561	0.6445	0.6431	FR
	3.5	0.7020	0.6732	0.6731	0.6631	FR
	4	0.7750	0.7142	0.6932	0.6069	FR
	4.5	0.7781	0.7244	0.6948	0.7022	MLE
60	2	0.6620	0.5628	0.7088	0.6996	MOM
	2.5	0.6720	0.6033	0.7033	0.6863	MOM
	3	0.6730	0.6469	0.7311	0.7732	MOM
	3.5	0.7020	0.6631	0.7421	0.7662	MOM
	4	0.7750	0.6702	0.7001	0.6645	MOM
	4.5	0.7781	0.6881	0.7088	0.7732	MOM
80	2	0.6620	0.5542	0.5526	0.6062	MLE
	2.5	0.6720	0.6063	0.6036	0.6244	MLE
	3	0.6730	0.6396	0.6242	0.6315	MLE
	3.5	0.7020	0.6642	0.6521	0.6221	FR
	4	0.7750	0.6824	0.6604	0.6332	FR
	4.5	0.7781	0.6943	0.6613	0.6562	FR

Table 4. values of $\tilde{h}(x)$ when (β=1.5,θ=0.5,$\tilde{k}$=0.2,R=1000)

n	x_i	$\tilde{h}(x_i)$	MOM $\tilde{h}(x_i)$	MLE $\tilde{h}(x_i)$	FR $\tilde{h}(x_i)$	Best
20	2	0.5503	0.6062	0.5718	0.5625	FR
	2.5	0.6020	0.6416	0.6223	0.6233	MLE
	3	0.6332	0.6726	0.6642	0.6456	FR

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	3.5	0.6527	0.7032	0.6562	0.6335	FR
	4	0.6884	0.7182	0.6632	0.6709	MLE
	4.5	0.7002	0.7242	0.6622	0.6886	MLE
40	2	0.5503	0.5872	0.5723	0.5862	MLE
	2.5	0.6020	0.6334	0.6331	0.6020	FR
	3	0.6332	0.6652	0.6642	0.6443	MLE
	3.5	0.6527	0.6884	0.6782	0.6630	FR
	4	0.6884	0.7032	0.6774	0.6680	FR
	4.5	0.7002	0.7182	0.7172	0.7211	MLE
60	2	0.5503	0.5686	0.5699	0.5638	FR
	2.5	0.6020	0.6288	0.6238	0.6132	FR
	3	0.6332	0.6514	0.6613	0.6649	MLE
	3.5	0.6527	0.6627	0.6721	0.6720	MOM
	4	0.6884	0.6921	0.6928	0.7631	MOM
	4.5	0.7002	0.7054	0.6774	0.7662	MOM
80	2	0.5503	0.5592	0.5663	0.5562	FR
	2.5	0.6020	0.6044	0.6003	0.6060	MLE
	3	0.6332	0.6322	0.6324	0.6392	MOM
	3.5	0.6527	0.6411	0.6621	0.6641	MOM
	4	0.6884	0.6521	0.6781	0.6362	MOM
	4.5	0.7002	0.6866	0.6632	0.6863	MLE

Table 5. values of $\tilde{h}(x)$ when (β=1.5,θ=0.5,$\tilde{k}$=0.5,R=1000)

n	x_i	$\tilde{h}(x_i)$	MOM $\tilde{h}(x_i)$	MLE $\tilde{h}(x_i)$	FR $\tilde{h}(x_i)$	Best
20	2	0.3358	0.3125	0.2866	0.3178	MLE
	2.5	0.3633	0.3506	0.3327	0.3862	MLE
	3	0.3662	0.3607	0.3604	0.4224	MLE
	3.5	0.3871	0.3961	0.4032	0.4413	MOM
	4	0.3874	0.4025	0.4152	0.4617	MOM
	4.5	0.4032	0.4465	0.4266	0.4885	MLE
40	2	0.3358	0.2988	0.2766	0.3286	MLE
	2.5	0.3633	0.3376	0.3225	0.3882	MLE
	3	0.3662	0.3634	0.3528	0.4242	MOM
	3.5	0.3871	0.3825	0.3657	0.4629	MOM
	4	0.3874	0.3978	0.3916	0.4772	MLE

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	4.5	0.4032	0.4192	0.4046	0.5012	MOM
60	2	0.3358	0.2899	0.2737	0.5113	MLE
	2.5	0.3633	0.3267	0.3166	0.5123	MLE
	3	0.3662	0.3542	0.3464	0.5129	MLE
	3.5	0.3871	0.3745	0.3692	0.5116	MLE
	4	0.3874	0.4019	0.3854	0.5332	FR
	4.5	0.4032	0.4268	0.3989	0.3662	FR
80	2	0.3358	0.2813	0.4156	0.4102	FR
	2.5	0.3633	0.3266	0.3952	0.4452	MLE
	3	0.3662	0.3492	0.3823	0.4668	MOM
	3.5	0.3871	0.3696	0.3810	0.4521	MOM
	4	0.3874	0.4072	0.7762	0.4551	MOM
	4.5	0.4032	0.4155	0.7751	0.4667	MOM

Table 6. values of $\tilde{h}(x)$ when (β=1.5,θ=0.8,$\tilde{k}$=0.2,R=1000)

n	x_i	$\tilde{h}(x_i)$	MOM $\tilde{h}(x_i)$	MLE $\tilde{h}(x_i)$	FR $\tilde{h}(x_i)$	Best
20	2	0.3778	0.3172	0.4958	0.2863	FR
	2.5	0.3882	0.3506	0.4376	0.3259	FR
	3	0.3807	0.3766	0.4662	0.3647	FR
	3.5	0.3948	0.3964	0.4230	0.4708	MOM
	4	0.4152	0.4107	0.4052	0.4922	MOM
	4.5	0.4256	0.4226	0.4011	0.4063	MLE
40	2	0.3778	0.4467	0.4869	0.4157	FR
	2.5	0.3882	0.4875	0.4282	0.4266	FR
	3	0.3807	0.4906	0.3562	0.4317	MLE
	3.5	0.3948	0.4992	0.3762	0.4377	MLE
	4	0.4152	0.4907	0.3904	0.4079	MLE
	4.5	0.4256	0.4292	0.4152	0.4192	MLE
60	2	0.3778	0.3866	0.4019	0.4262	MOM
	2.5	0.3882	0.3667	0.4118	0.4193	MOM
	3	0.3807	0.3543	0.4119	0.4266	MOM
	3.5	0.3948	0.3743	0.4267	0.4252	MOM
	4	0.4152	0.4015	0.4268	0.4223	MOM
	4.5	0.4256	0.4119	0.4326	0.4202	MOM
	2	0.3778	0.4042	0.3277	0.2833	FR

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80	2.5	0.3882	0.3851	0.3776	0.3236	FR
	3	0.3807	0.3692	0.3952	0.3521	FR
	3.5	0.3948	0.3659	0.4056	0.3719	MOM
	4	0.4152	0.3743	0.4152	0.4185	MOM
	4.5	0.4256	0.3802	0.4671	0.4206	MOM

Table 7. values of $\tilde{h}(x)$ when (β=1.5,θ=0.8,$\tilde{k}$=0.5,R=1000)

Conclusion

The fuzziness in hazard function was caused due incomplete data, also imperfect information, so we introduce fuzzy factor ($\tilde{k}$) for comparing different three estimators for hazard rate function.

MOM is dominated with percentage ($\frac{44}{144} = 39.56\%$), MLE dominated with percentage ($\frac{42}{144} = 29.25\%$), and FR dominated with ($\frac{58}{144} = 40.28\%$).

We notice that hazard rate function estimated by frequency ratio is dominated with percentage (40.28%), then MOM, and finally MLE.

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